

**CBSE EXAMINATION PAPER—2019 (DELHI)**  
**MATHEMATICS**  
**Class XII**

*Time allowed : 3 hrs.*

*Maximum Marks : 100*

**GENERAL INSTRUCTIONS:**

- (i) **All** questions are compulsory.
- (ii) This question paper consists of **29** questions divided into four sections A, B, C and D. Section A comprises of 4 questions of **one mark** each, Section B comprises of 8 questions of **two marks** each, Section C comprises of 11 questions of **four marks** each and Section D comprises of 6 questions of **six marks** each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in **1** question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only **one** of the alternatives in all such questions.
- (v) Use of calculators is **not** permitted. You may ask logarithmic tables, if required.

**Set-I**

**SECTION A**

Question numbers 1 to 4 carry 1 mark each.

1. If A and B are square matrices of the same order 3, such that  $|A| = 2$  and  $AB = 2I$ , write the value of  $|B|$ .
2. If  $f(x) = x + 1$ , find  $\frac{d}{dx} (f \circ f)(x)$ .
3. Find the order and the degree of the differential equation  $x^2 \frac{d^2 y}{dx^2} = \left\{ 1 + \left( \frac{dy}{dx} \right)^2 \right\}^4$ .
4. If a line makes angles  $90^\circ$ ,  $135^\circ$ ,  $45^\circ$  with the  $x$ ,  $y$ , and  $z$  axes respectively, find its direction cosines.

**OR**

Find the vector equation of the line which passes through the point  $(3, 4, 5)$  and is parallel to the vector  $2\hat{i} + 2\hat{j} - 3\hat{k}$ .

## SECTION B

Question numbers 5 to 12 carry 2 marks each.

5. Examine whether the operation  $*$  defined on  $\mathbb{R}$  by  $a * b = ab + 1$  is (i) a binary or not. (ii) if a binary operation, is it associative or not?

6. Find a matrix  $A$  such that  $2A - 3B + 5C = O$ , where  $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$ .

7. Find:  $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$ .

8. Find:  $\int \sqrt{1 - \sin 2x} dx$   $\frac{\pi}{4} < x < \frac{\pi}{2}$ .

OR

Find:  $\int \sin^{-1}(2x) dx$ .

9. Form the differential equation representing the family of curves  $y = e^{2x}(a + bx)$ , where 'a' and 'b' are arbitrary constants.
10. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is  $\sqrt{3}$ .

OR

If  $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$  and  $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$ , find  $[\vec{a} \vec{b} \vec{c}]$

11. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.
12. A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes? (ii) atmost 5 successes?

OR

The random variable X has a probability distribution P(X) of the following form, where 'k' is the some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

## SECTION-C

Question numbers 13 to 23 carry 4 marks each.

13. Show that the relation R on  $\mathbb{R}$  defined as  $R = \{(a, b) : a \leq b\}$ , is reflexive, and transitive but not symmetric.

**OR**

Prove that the function  $f : \mathbb{N} \rightarrow \mathbb{N}$ , defined by  $f(x) = x^2 + x + 1$  is one-one but not onto. Find inverse of  $f : \mathbb{N} \rightarrow \mathbb{S}$ , where  $\mathbb{S}$  is range of  $f$ .

14. Solve:  $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$ .

15. Using properties of determinants, prove that 
$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3.$$

16. If  $\log(x^2 + y^2) = 2 \tan^{-1} \left( \frac{y}{x} \right)$ , show that  $\frac{dy}{dx} = \frac{x + y}{x - y}$ .

**OR**

If  $x^y - y^x = a^b$ , find  $\frac{dy}{dx}$ .

17. If  $y = (\sin^{-1} x)^2$ , prove that  $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$ .

18. Find the equation of tangent to the curve  $y = \sqrt{3x - 2}$  which is parallel to the line  $4x - 2y + 5 = 0$ . Also, write the equation of normal to the curve at the point of contact.

19. Find:  $\int \frac{3x + 5}{x^2 + 3x - 18} dx$ .

20. Prove that  $\int_0^a f(x) dx = \int_0^a f(a - x) dx$ , hence evaluate  $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ .

21. Solve the differential equation:  $x dy - y dx = \sqrt{x^2 + y^2} dx$ , given that  $y = 0$  when  $x = 1$ .

**OR**

Solve the differential equation:  $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ , subject to the initial condition  $y(0) = 0$ .

22. If  $\hat{i} + \hat{j} + \hat{k}$ ,  $2\hat{i} + 5\hat{j}$ ,  $3\hat{i} + 2\hat{j} - 3\hat{k}$  and  $\hat{i} - 6\hat{j} - \hat{k}$  respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether  $\vec{AB}$  and  $\vec{CD}$  are collinear or not.

23. Find the value of  $\lambda$ , so that the lines  $\frac{1 - x}{3} = \frac{7y - 14}{\lambda} = \frac{z - 3}{2}$  and  $\frac{7 - 7x}{3\lambda} = \frac{y - 5}{1} = \frac{6 - z}{5}$  are at right angles. Also, find whether the lines are intersecting or not.

### SECTION D

Question numbers 24 to 29 carry 4 marks each.

24. If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ , find  $A^{-1}$ . Hence, solve the system of equations

$$x + y + z = 6, x + 2z = 7, 3x + y + z = 12.$$

**OR**

Find the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

25. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is  $8 \text{ m}^3$ . If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank?
26. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

**OR**

Find the area of the region lying above  $x$ -axis and included between the circle  $x^2 + y^2 = 8x$  and inside of the parabola  $y^2 = 4x$ .

27. Find the vector and Cartesian equations of the plane passing through the points (2, 2, -1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

**OR**

Find the vector equation of the plane that contains the lines  $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$  and the point (-1, 3, -4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained.

28. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A?
29. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.

## Set-II

Questions which are different from Set-I.

Question numbers 1 to 4 carry 1 mark each.

### SECTION-A

2. If  $f(x) = x + 7$  and  $g(x) = x - 7$ ,  $x \in \mathbb{R}$ , then find  $\frac{d}{dx}(\text{fog})(x)$ .

3. Find the value of  $x - y$ , if  $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ .

### SECTION-B

Question numbers 5 to 12 carry 2 marks each.

6. If  $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ , then find  $(A^2 - 5A)$ .

12. Find:  $\int \frac{\tan^2 x \sec^2 x}{1 - \tan^6 x} dx$ .

### SECTION-C

Question numbers 13 to 23 carry 4 marks each.

13. Solve for  $x$ :  $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ .

18. Using properties of determinants, prove the following:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

19. If  $x = \cos t + \log \tan \left(\frac{t}{2}\right)$ ,  $y = \sin t$ , then find the values of  $\frac{d^2y}{dt^2}$  and  $\frac{d^2y}{dx^2}$  at  $t = \frac{\pi}{4}$ .

### SECTION-D

Question numbers 24 to 29 carry 6 marks each.

24. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius  $r$  is  $\frac{4r}{3}$ . Also find the maximum volume of cone.

25. If  $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ , then find  $A^{-1}$ . Hence solve the following system of equations:

$$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3.$$

**OR**

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

### Set-III

Questions which are different from Set-I and Set-II.

#### SECTION-A

Question numbers 1 to 4 carry 1 mark each.

1. If  $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ , then find the matrix A.

2. Write the order and the degree of the following differential equation:

$$x^3 \left( \frac{d^2 y}{dx^2} \right)^2 + x \left( \frac{dy}{dx} \right)^4 = 0$$

#### SECTION-B

Question numbers 5 to 12 carry 2 marks each.

5. Find:  $\int \sin x \cdot \log \cos x \, dx$ .

6. Evaluate:  $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx$

**OR**

Evaluate:  $\int_{-1}^2 \frac{|x|}{x} \, dx$ .

#### SECTION-C

Question numbers 13 to 23 carry 4 marks each.

12. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$$

20. Find:  $\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} \, dx$ .

21. Solve the differential equation:  $\frac{dy}{dx} - \frac{2x}{1+x^2}y = x^2 + 2$

**OR**

Solve the differential equation:  $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1; y(0) = 0.$

**SECTION-D**

*Question numbers 24 to 29 carry 6 marks each.*

26. Prove that the curves  $y^2 = 4x$  and  $x^2 = 4y$  divide the area of the square bounded by sides  $x = 0, x = 4, y = 4$  and  $y = 0$  into three equal parts.

**OR**

Using integration, find the area of the triangle whose vertices are  $(2, 3), (3, 5)$  and  $(4, 4)$ .

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.