CBSE EXAMINATION PAPER—2019 (DELHI)
MATHEMATICS
CLASS–X

Time : 3 hrs. *Max. Marks : 100*

GENERAL INSTRUCTIONS:

(*i*) ***All*** *questions are compulsory.*

(*ii*) *The question paper consists of* ***29*** *questions divided into four sections A, B, C and D. Section A comprises of* ***4*** *questions of* ***one mark*** *each, Section B comprises of 8 questions of* ***two marks*** *each, Section C comprises of* ***11*** *questions of* ***four marks*** *each and Section D comprises of* ***6*** *questions of* ***six marks*** *each.*

(*iii*) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(*iv*) *There is no overall choice. However, internal choice has been provided in* ***1*** *question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is* ***not*** *permitted. You may ask for logarithmic tables, if required.*

Set–I

SECTION-A

Question numbers 1 to 4 carry 1 mark each.

1. Form the differential equation representing the family of curves $y = \frac{A}{x} + 5$, by eliminating the arbitrary constant A.

2. If A is a square matrix of order 3, with | A | = 9, then write the value of | 2. adj A |.

3. Find the acute angle between the planes $\vec{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$ and $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$.

OR

Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

4. If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

SECTION-B

Question numbers 5 to 12 carry 2 marks each.

5. Find: $\int_{-\frac{\pi}{4}}^{0} \frac{1 + \tan x}{1 - \tan x} dx$.

6. Let $*$ be an operation defined as $* : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, such that $a * b = 2a + b$, $a, b \in \mathbb{R}$. Check if $*$ is a binary operation. If yes, find if it is associative too.

7. X and Y are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally.

OR

Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

8. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB – BA is a skew symmetric matrix.

9. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

10. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

OR

In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

11. Solve the following differential equation:

$$\frac{dy}{dx} + y = \cos x - \sin x$$

12. Find: $\int x \cdot \tan^{-1} x \, dx$

OR

Find: $\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$

SECTION-C

Question numbers 13 to 23 carry 4 marks each.

13. Using properties of determinants, find the value of x for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0.$$

14. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that $y = 1$ when $x = 0$.

15. Let A = R – {2} and B = R – {1}. If f : A → B is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

OR

Show that the relation S in the Set A = $\{x \in Z : 0 \le x \le 12\}$ given by S = $\{(a, b) : a, b \in Z, |a - b|$ is divisible by 3$\}$ is an equivalence relation.

16. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ w.r.t. x.

17. If $x = \sin t$, $y = \sin pt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

OR

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$ with respect to $\cos^{-1} x^2$.

18. Prove that:

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$$

19. If $y = (x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$.

20. Prove that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$, and hence evaluate $\int_0^1 x^2(1-x)^n\, dx$.

21. Find the value of x, for which the four points A(x, – 1, – 1), B(4, 5, 1), C(3, 9, 4) and D(– 4, 4, 4) are coplanar.

22. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

23. Find the vector equation of the plane determined by the points A(3, – 1, 2), B(5, 2, 4) and C(– 1, – 1, 6). Hence, find the distance of the plane, thus obtained, from the origin.

SECTION-D

Question numbers 24 to 29 carry 6 marks each.

24. Using integration, find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

25. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist?

26. Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$.

OR

Using matrices, solve the following system of linear equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

27. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

OR

Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$.

28. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. It costs ₹ 50 per kg to produce food I. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C and it costs ₹ 70 per kg to produce food II. Formulate this problem as a LPP to minimise the cost of a mixture that will produce the required diet. Also find the minimum cost.

29. Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.

OR

Find the coordinates of the foot of the perpendicular Q drawn from P(3,2, 1) to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.

Set–II

Questions different from Set-I.

SECTION-A

3. If $y = \text{cosec}\,(\cot\sqrt{x})$, then find $\frac{dy}{dx}$.

4. Write the integrating factor of the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)\, dx$.

SECTION-C

21. Using properties of determinants, find the value of k if

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k(x^3 + y^3)$$

22. Find: $\int \frac{1}{\sin(x-a)\cos(x-b)}\, dx$.

23. Prove that: $\sin^{-1}\left(\frac{8}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \cot^{-1}\left(\frac{36}{77}\right)$

SECTION-D

28. Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

29. There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tails 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

Questions different from Set I and Set II.

Set–III

SECTION-A

3. A is a square matrix with $|A| = 4$. Then find the value of $|A.(\text{adj } A)|$.

SECTION-B

Question number 5 to 12 carry 2 marks each.

11. The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

SECTION-C

13. A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

21. Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

22. Find the value of x such that the four points with position vectors, $A(3\hat{i} + 2\hat{j} + \hat{k})$, $B(4\hat{i} + x\hat{j} + 5\hat{k})$, $C(4\hat{i} + 2\hat{j} - 2\hat{k})$ and $D(6\hat{i} + 5\hat{j} - \hat{k})$ are coplanar.

23. If $y = (\log x)^x + x^{\log x}$, find $\frac{dy}{dx}$.

SECTION-D

29. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B, to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates each machine for atmost 12 hours a day ? Convert it into an LPP and solve graphically.